Titre :	Dynamic routing through quantum repeaters

Quantum networks are needed to achieve the promises of quantum communications: cryptographic applications, distributed quantum computing, distributed measurement of physical quantities, delegated secure quantum computing, or even distribution of resource states inside a quantum computer. Investigations in both elementary networks components — sources, links repeaters, etc. — and ways to verify and use quantum resource states — Bell pairs, GHZ, graph states — for high level protocols are an important part of quantum information research since its origin in the 1990s, but the study of actual networking problems, like routing, in quantum networks themselves only started a few years ago [2–6] but is now a blooming part of quantum technologies. Our aim with this thesis is to study and optimize the simultaneous routing of multiple bipartite entangled links in quantum networks based on first generation quantum repeaters, combining the expertise of the two supervisors — A. Giovanidis in classical routing for telecommunication networks with emphasis in wireless [A, B] and F. Grosshans in the distribution of multipartite states over quantum networks [C, D].

A proposed solution to overcome the challenge of long distance entanglement distribution is the use of so-called first generation quantum repeaters [1], which combine quantum memory and entanglement swapping to “glue” together two short entangled links into a longer link.This idea can be generalised to a network where each node can be adjacent to three links or more, in which case it can operate as a switch and decide which links to glue together; this is a routing decision. A switch will further be equipped quantum memories to locally store unserved qubit pairs. A quantum network should provide service to multiple origin-destination (o-d) pairs simultaneously, each o-d pair requiring some rate of end-to-end entanglement. In such general cases, the switch will also decide which o-d pair to serve over what route. To achieve this, links of the network can be time-shared among several paths of o-d pairs, which renders routing decisions very challenging.

This fresh and important problem of entanglement routing over a quantum network combines quantum physics with computer science and telecommunication networks; it has started gaining a lot of attention. The authors in [2,3] analyse the performance of a quantum switch, whereas in [4] the authors propose a heuristic dynamic routing protocol. The authors in [5] propose greedy and best-effort distributed routing algorithms for multiple o-d pairs, where nodes can use only local information. Furthermore, the authors in [6] borrow tools from classical networks, to optimise the achievable EPR-pair average rate between multiple source-destination pairs in a quantum Internet, using linear programming.

The above works have made the first steps towards the proposal of efficient and practical quantum routing protocols. Efficiency is understood here as rate maximising or delay minimising. There is however lots of room for improvement. Specifically, the above works can precisely analyse the dynamic performance of a single switch under a given simple routing policy [2,3], or they propose heuristic routing policies that work good-enough [5], or optimise the network routing on average. But, more precise results for dynamic quantum routing through repeaters is missing.

We aim to analyse dynamic quantum routing for arbitrary topologies and multiple o-d pairs through the use of Markov chains (discrete or continuous), which are sufficiently expressive to describe the network dynamics over time, and whose application is standard for dynamic control problems. More specifically, we want to borrow existing classical tools to derive optimal dynamic quantum routing protocols for any required utility metric, exploiting Markovian drift minimisation [7, 8], a topic on which A. Giovanidis has considerable experience for classical wireless networks [A, B]. To do so, an important feature to explore are the quantum memories per node, which can store more than one photon pair, until it potentially is exploited to teleport a qubit or get lost due to decogerence. We will thus build our analysis on the concept of queues of entangled pairs, whereas the routing decisions are taken over the queues’ servers. Obviously, the role of memory and the lifetime of photons on a node will determine the protocol routing decisions, and resulting performance.

E.g. short-memory networks will possibly route greedily and myopically, whereas longer-memory networks can decide on the entanglement swaps by looking further ahead in the future.

Interestingly, service can include not only bipartite entanglement swaps, but also three- or in general n-qubit entanglement [C, D]; these can generalise routing to include more efficient protocols and/or more generic distribution tasks. A major challenge is the amount of local information necessary to allow the protocol to perform optimally. We intend to begin by deriving optimal protocols with centrally available information and continue by studying the performance degradation, when information is limited on a per node basis. Finally, an important challenge is to adapt the suggestion for dynamic protocols to limitations of real-platform implementations.

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